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FOUR YEAR B.Sc. (Honours) DEGREE EXAMINATION, APRIL/MAY 2025.

FOURTH SEMESTER

Mathematics – Minor

Paper 4 – INTRODUCTION TO REAL ANALYSIS

(w.e.f. 2023–24 Regulations)

Time : Three hours

Maximum : 70 marks

(No additional sheet will be supplied)

SECTION A — (5 × 4 = 20 marks)

Answer any FIVE questions.

1. Show that every convergent sequence is bounded. Give an example to show the Converse is not true.

2. If $S_n = \sqrt{n+1} - \sqrt{n}$, then show that $\lim S_n = 0$.

3. Test the convergence $\sum_{n=1}^{\infty} \sqrt{n+1} - \sqrt{n}$.

4. Test the convergence $\sum \frac{n!}{n^n}$.

5. If $f(x) = \frac{1 - e^{1/x}}{1 + e^{1/x}}$ when $x \neq 0$ find $\lim_{x \to 0} f(x)$.

6. Examine the continuity of the function $f(x) = \frac{1 - \cos x}{x^2}, x \neq 0; f(0) = 1$ at $x = 0$.

7. Show that $f(x) = |x - 2|$ is not derivable at $x = 2$.

8. Discuss the applicability of Rolle's theorem on $f(x) = x^2 + 6x^2 + 11x - 6, a = 1, b = 3$.

9. If $f(x) = x$ on [0, 1] and $P = \{0, 1/3, 2/3\}$ find $U(P, f)$ and $L(P, f)$.

10. If $f(x) = k. \forall x \in [a, b]$ prove that $f \in R[a, b]$.

SECTION B — (5 × 10 = 50 marks)

Answer ALL the following questions.

11. Show that a monotonic sequence is convergent iff it is bounded.

Or

12. State and prove Cauchy's general principle convergence.

13. Test the convergence (a) $\sum\left(1+\frac{1}{n}\right)^{-n2}$ (b) $\sum\left(\frac{n}{n+1}\right)^{n2}$.

Or

14. State and Prove D'Alembert's ratio test.

15. If f is continuous on $[a, b]$ then show that f is bounded on $[a, b]$ and attains its bounds.

Or

16. If a function f is continuous on $[a, b]$ then it is uniformly continuous on $[a, b]$.

17. State and prove Lagrange's mean value theorem.

Or

18. Find 'c' of Cauchy's mean value theorem for $f(x)=\sqrt{x}$ and $g(x)=1/\sqrt{x}$ in $[a, b]$ where $0<a<b$.

19. State and prove fundamental theorem of integral calculus.

Or

20. If $f:[a,b]\to R$ is monotonic on $[a, b]$, then prove that f is integrable on $[a, b]$.
